1. ELECTROWEAK MODEL AND CONSTRAINTS ON NEW PHYSICS

Revised December 2011 by J. Erler (U. Mexico and Institute for Advanced Study) and P. Langacker (Princeton University and Institute for Advanced Study).

1.1. Introduction

The standard model of the electroweak interactions (SM) [1] is based on the gauge group $SU(2) \times U(1)$, with gauge bosons W^i_μ, $i = 1, 2, 3$, and B_μ for the SU(2) and U(1) factors, respectively, and the corresponding gauge coupling constants g and g'. The left-handed fermion fields of the i^{th} fermion family transform as doublets $\Psi_i = \begin{pmatrix} \nu_i \\ \ell^-_i \end{pmatrix}$ and $\begin{pmatrix} u_i \\ d'_i \end{pmatrix}$ under SU(2), where $d'_i \equiv \sum_j V_{ij}\, d_j$, and V is the Cabibbo-Kobayashi-Maskawa mixing matrix. (Constraints on V and tests of universality are discussed in Ref. 2 and in the Section on "The CKM Quark-Mixing Matrix". The extension of the formalism to allow an analogous leptonic mixing matrix is discussed in the Section on "Neutrino Mass, Mixing, and Oscillations".) The right-handed fields are SU(2) singlets. In the minimal model there are three fermion families.

A complex scalar Higgs doublet, $\phi \equiv \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}$, is added to the model for mass generation through spontaneous symmetry breaking with potential* given by,

$$V(\phi) = \mu^2 \phi^\dagger \phi + \frac{\lambda^2}{2} \left(\phi^\dagger \phi\right)^2 . \tag{1.1}$$

For μ^2 negative, ϕ develops a vacuum expectation value, $v/\sqrt{2}$, where $v \approx 246.22$ GeV, breaking part of the electroweak (EW) gauge symmetry, after which only one neutral Higgs scalar, H, remains in the physical particle spectrum. In non-minimal models there are additional charged and neutral scalar Higgs particles [3].

After the symmetry breaking the Lagrangian for the fermion fields, ψ_i, is

$$\begin{aligned} \mathscr{L}_F &= \sum_i \overline{\psi}_i \left(i \not\partial - m_i - \frac{g m_i H}{2 M_W} \right) \psi_i \\ &- \frac{g}{2\sqrt{2}} \sum_i \overline{\Psi}_i \, \gamma^\mu \left(1 - \gamma^5\right) \left(T^+ W^+_\mu + T^- W^-_\mu\right) \Psi_i \\ &- e \sum_i q_i \, \overline{\psi}_i \, \gamma^\mu \, \psi_i \, A_\mu \\ &- \frac{g}{2\cos\theta_W} \sum_i \overline{\psi}_i \, \gamma^\mu \left(g^i_V - g^i_A \gamma^5\right) \psi_i \, Z_\mu \, . \end{aligned} \tag{1.2}$$

---

* There is no generally accepted convention to write the quartic term. Our numerical coefficient simplifies Eq. (1.3*a*) below and the squared coupling preserves the relation between the number of external legs and the power counting of couplings at a given loop order. This structure also naturally emerges from physics beyond the SM, such as supersymmetry.

$\theta_W \equiv \tan^{-1}(g'/g)$ is the weak angle; $e = g \sin\theta_W$ is the positron electric charge; and $A \equiv B\cos\theta_W + W^3 \sin\theta_W$ is the photon field (γ). $W^{\pm} \equiv (W^1 \mp iW^2)/\sqrt{2}$ and $Z \equiv -B\sin\theta_W + W^3\cos\theta_W$ are the charged and neutral weak boson fields, respectively. The Yukawa coupling of H to ψ_i in the first term in $\mathscr{L}_F$, which is flavor diagonal in the minimal model, is $gm_i/2M_W$. The boson masses in the EW sector are given (at tree level, *i.e.*, to lowest order in perturbation theory) by,

$$M_H = \lambda\, v, \tag{1.3a}$$

$$M_W = \frac{1}{2}\, g\, v = \frac{e\, v}{2\sin\theta_W}, \tag{1.3b}$$

$$M_Z = \frac{1}{2}\sqrt{g^2 + g'^2}\, v = \frac{e\, v}{2\sin\theta_W \cos\theta_W} = \frac{M_W}{\cos\theta_W}, \tag{1.3c}$$

$$M_\gamma = 0. \tag{1.3d}$$

The second term in $\mathscr{L}_F$ represents the charged-current weak interaction [4–7], where T^+ and T^- are the weak isospin raising and lowering operators. For example, the coupling of a W to an electron and a neutrino is

$$-\frac{e}{2\sqrt{2}\sin\theta_W}\left[W_\mu^- \,\overline{e}\,\gamma^\mu \left(1-\gamma^5\right)\nu + W_\mu^+ \,\overline{\nu}\,\gamma^\mu \left(1-\gamma^5\right) e\right]. \tag{1.4}$$

For momenta small compared to M_W, this term gives rise to the effective four-fermion interaction with the Fermi constant given by $G_F/\sqrt{2} = 1/2v^2 = g^2/8M_W^2$. CP violation is incorporated into the EW model by a single observable phase in V_{ij}.

The third term in $\mathscr{L}_F$ describes electromagnetic interactions (QED) [8–10], and the last is the weak neutral-current interaction [5–7]. The vector and axial-vector couplings are

$$g_V^i \equiv t_{3L}(i) - 2q_i \sin^2\theta_W, \tag{1.5a}$$

$$g_A^i \equiv t_{3L}(i), \tag{1.5b}$$

where $t_{3L}(i)$ is the weak isospin of fermion i ($+1/2$ for u_i and ν_i; $-1/2$ for d_i and e_i) and q_i is the charge of ψ_i in units of e.

The first term in Eq. (1.2) also gives rise to fermion masses, and in the presence of right-handed neutrinos to Dirac neutrino masses. The possibility of Majorana masses is discussed in the Section on "Neutrino Mass, Mixing, and Oscillations".

1.2. Renormalization and radiative corrections

In addition to the Higgs boson mass, M_H, the fermion masses and mixings, and the strong coupling constant, α_s, the SM has three parameters. A particularly useful set contains the Z mass, the Fermi constant, and the fine structure constant, which will be discussed in turn:

The Z boson mass, $M_Z = 91.1876 \pm 0.0021$ GeV, has been determined from the Z lineshape scan at LEP 1 [11].

The Fermi constant, $G_F = 1.1663787(6) \times 10^{-5}$ GeV^{-2}, is derived from the muon lifetime formula***,

$$\frac{\hbar}{\tau_\mu} = \frac{G_F^2 m_\mu^5}{192\pi^3} F(\rho) \left[1 + H_1(\rho) \frac{\widehat{\alpha}(m_\mu)}{\pi} + H_2(\rho) \frac{\widehat{\alpha}^2(m_\mu)}{\pi^2} \right], \quad (1.6)$$

where $\rho = m_e^2/m_\mu^2$, and where

$$F(\rho) = 1 - 8\rho + 8\rho^3 - \rho^4 - 12\rho^2 \ln\rho = 0.99981295, \quad (1.7a)$$

$$H_1(\rho) = \frac{25}{8} - \frac{\pi^2}{2} - \left(9 + 4\pi^2 + 12\ln\rho\right)\rho + 16\pi^2\rho^{3/2} + \mathcal{O}\left(\rho^2\right) = -1.80793, \quad (1.7b)$$

$$H_2(\rho) = \frac{156815}{5184} - \frac{518}{81}\pi^2 - \frac{895}{36}\zeta(3) + \frac{67}{720}\pi^4 + \frac{53}{6}\pi^2 \ln 2 - (0.042 \pm 0.002)_{\text{had}} - \frac{5}{4}\pi^2\sqrt{\rho} + \mathcal{O}(\rho) = 6.64, \quad (1.7c)$$

$$\widehat{\alpha}\left(m_\mu\right)^{-1} = \alpha^{-1} + \frac{1}{3\pi}\ln\rho + \mathcal{O}(\alpha) = 135.901 \quad (1.7d)$$

The massless corrections to H_1 and H_2 have been obtained in Refs. 13 and 14, respectively, where the term in parentheses is from the hadronic vacuum polarization [14]. The mass corrections to H_1 have been known for some time [15], while those to H_2 are more recent [16]. Notice the term linear in m_e whose appearance was unforeseen and can be traced to the use of the muon pole mass in the prefactor [16]. The remaining uncertainty in G_F is experimental and has recently been reduced by an order of magnitude by the MuLan collaboration [12] at the PSI.

The fine structure constant, $\alpha = 1/137.035999074(44)$, is currently dominated by the $e^\pm$ anomalous magnetic moment [10].

Further free parameters entering into Eq. (1.2) are the quark and lepton masses, where m_i is the mass of the i^{th} fermion ψ_i. For the quarks these are the current masses. For the light quarks, as described in the note on "Quark Masses" in the Quark Listings, $\widehat{m}_u = 2.5^{+0.6}_{-0.8}$ MeV, $\widehat{m}_d = 5.0^{+0.7}_{-0.9}$ MeV, and $\widehat{m}_s = 100^{+30}_{-20}$ MeV. These are running $\overline{\text{MS}}$ masses evaluated at the scale $\mu = 2$ GeV. For the heavier quarks we use QCD sum rule [53] constraints [54] and recalculate their masses in each call of our fits to account for their direct α_s dependence. We find, $\widehat{m}_c(\mu = \widehat{m}_c) = 1.267^{+0.032}_{-0.040}$ GeV and $\widehat{m}_b(\mu = \widehat{m}_b) = 4.197 \pm 0.025$ GeV, with a correlation of 24%.

The top quark "pole" mass (the quotation marks are a reminder that quarks do not form asymptotic states), $m_t = 173.4 \pm 0.9$ GeV, is an average of published and preliminary CDF and DØ results from run I and II [56] with first results by the CMS [57] and ATLAS [58]

*** In the spirit of the Fermi theory, we incorporated the small propagator correction, $3/5\ m_\mu^2/M_W^2$, into Δr (see below). This is also the convention adopted by the MuLan collaboration [12]. While this breaks with historical consistency, the numerical difference was negligible in the past.

collaborations averaged in ignoring correlations. To gauge the possible impact of the neglect of correlations involving the LHC experiments, we also averaged the results conservatively assuming that the entire 0.75 GeV systematic of the Tevatron average is fully correlated with a 0.75 GeV component in both CMS and ATLAS. Incidentally, this yields correlations of similar size as those between the two Tevatron experiments and the two Runs and reduces the central value by 0.15 GeV. Within round-off we expect a more refined average to coincide with ours. Our average differs slightly from the value, $m_t = 173.5 \pm 0.6 \pm 0.8$ GeV, which appears in the top quark Listings in this *Review* and which is based exclusively on published results. We are working, however, with $\overline{\text{MS}}$ masses in all expressions to minimize theoretical uncertainties. Such a short distance mass definition (unlike the pole mass) is free from non-perturbative and renormalon [59] uncertainties. We therefore convert to the top quark $\overline{\text{MS}}$ mass,

$$\widehat{m}_t\,(\mu = \widehat{m}_t) = m_t \left[1 - \frac{4}{3}\frac{\alpha_s}{\pi} + \mathcal{O}\left(\alpha_s^2\right)\right], \tag{1.8}$$

using the three-loop formula [60]. This introduces an additional uncertainty which we estimate to 0.5 GeV (the size of the three-loop term) and add in quadrature to the experimental pole mass error. This is convenient because we use the pole mass as an external constraint while fitting to the $\overline{\text{MS}}$ mass. We are assuming that the kinematic mass extracted from the collider events corresponds within this uncertainty to the pole mass. Using the BLM optimized [61] version of the two-loop perturbative QCD formula [62] (as we did in previous editions of this *Review*) gives virtually identical results. In summary, we will use $m_t = 173.4 \pm 0.9\,(\text{exp.}) \pm 0.5\,(\text{QCD})$ GeV $= 173.4 \pm 1.0$ GeV (together with $M_H = 117$ GeV) for the numerical values quoted in Sec. 1.2–Sec. 1.4.

$\sin^2\theta_W$ and M_W can be calculated from M_Z, $\widehat{\alpha}(M_Z)$, and G_F, when values for m_t and M_H are given; conversely (as is done at present), M_H can be constrained by $\sin^2\theta_W$ and M_W. The value of $\sin^2\theta_W$ is extracted from neutral-current processes (see Sec. 10.3) and Z pole observables (see Sec. 10.4) and depends on the renormalization prescription. There are a number of popular schemes [63–70] leading to values which differ by small factors depending on m_t and M_H, including the $\overline{\text{MS}}$ definition $\widehat{s}^{\,2}_Z$ and the on-shell definition $s^2_W \equiv 1 - M^2_W/M^2_Z$.

Experiments are at such level of precision that complete $\mathcal{O}(\alpha)$ radiative corrections must be applied. These are discussed in the full edition of this *Review*. A variety of related cross-section and asymmetry formulae are also discussed there.

1.3.1. *W and Z decays*:

The partial decay width for gauge bosons to decay into massless fermions $f_1\overline{f}_2$ (the numerical values include the small EW radiative corrections and final state mass effects) is given by

$$\Gamma\left(W^+ \to e^+\nu_e\right) = \frac{G_F M_W^3}{6\sqrt{2}\pi} \approx 226.36 \pm 0.05 \text{ MeV}\ , \tag{1.38a}$$

$$\Gamma\left(W^+ \to u_i\overline{d}_j\right) = \frac{C G_F M_W^3}{6\sqrt{2}\pi}\,|V_{ij}|^2 \approx 706.34 \pm 0.16 \text{ MeV}\ |V_{ij}|^2, \tag{1.38b}$$

$$\Gamma\left(Z \to \psi_i \overline{\psi}_i\right) = \frac{C G_F M_Z^3}{6\sqrt{2}\pi}\left[g_V^{i2} + g_A^{i2}\right] \approx \begin{cases} 167.22 \pm 0.01 \text{ MeV} \;\; (\nu\overline{\nu}) \,, \\ 84.00 \pm 0.01 \text{ MeV} \;\; \left(e^+ e^-\right) \,, \\ 300.26 \pm 0.05 \text{ MeV} \;\; (u\overline{u}, c\overline{c}) \\ 383.04 \pm 0.05 \text{ MeV} \;\; (d\overline{d}) \,, \\ 375.98 \mp 0.03 \text{ MeV} \;\; (b\overline{b}) \,. \end{cases} \tag{1.38}$$

For leptons $C = 1$, while for quarks

$$C = 3\left[1 + \frac{\alpha_s\left(M_V\right)}{\pi} + 1.409\,\frac{\alpha_s^2}{\pi^2} - 12.77\,\frac{\alpha_s^3}{\pi^3} - 80.0\,\frac{\alpha_s^4}{\pi^4}\right], \tag{1.39}$$

where the 3 is due to color and the factor in brackets represents the universal part of the QCD corrections [176] for massless quarks [177]. The $\mathcal{O}(\alpha_s^4)$ contribution in Eq. (1.39) is recent [178]. The $Z \to f\overline{f}$ widths contain a number of additional corrections: which are different for vector and axial-vector partial widths and are included through order α_s^3 and $\widehat{m}_q^4(M_Z^2)$ unless they are tiny; and singlet contributions starting from two-loop order which are large, strongly top quark mass dependent, family universal, and flavor non-universal [181]. The QED factor $1 + 3\alpha q_f^2/4\pi$, as well as two-loop order $\alpha\alpha_s$ and α^2 self-energy corrections [182] are also included. Working in the on-shell scheme, *i.e.*, expressing the widths in terms of $G_F M_{W,Z}^3$, incorporates the largest radiative corrections from the running QED coupling [63,183]. EW corrections to the Z widths are then incorporated by replacing $g_{V,A}^{i2}$ by the effective couplings $\overline{g}_{V,A}^{i2}$ defined in Eq. (1.36) of the full *Review*. Hence, in the on-shell scheme the Z widths are proportional to $1 + \rho_t$, where $\rho_t = 3G_F\, m_t^2/8\sqrt{2}\pi^2$. The $\overline{\text{MS}}$ normalization accounts also for the leading EW corrections [68]. There is additional (negative) quadratic m_t dependence in the $Z \to b\overline{b}$ vertex corrections [184] which causes $\Gamma(b\overline{b})$ to decrease with m_t. The dominant effect is to multiply $\Gamma(b\overline{b})$ by the vertex correction $1 + \delta\rho_{b\overline{b}}$, where $\delta\rho_{b\bar{b}} \sim 10^{-2}(-\frac{1}{2}\frac{m_t^2}{M_Z^2} + \frac{1}{5})$. In practice, the corrections are included in $\overline{g}_{V,A}^{\,b}$, as discussed in Sec. 1.2.

For three fermion families the total widths are predicted to be

$$\Gamma_Z \approx 2.4960 \pm 0.0002 \text{ GeV}\,, \qquad \Gamma_W \approx 2.0915 \pm 0.0005 \text{ GeV}\,. \tag{1.40}$$

We have assumed $\alpha_s(M_Z) = 0.1200$. An uncertainty in α_s of ± 0.002 introduces an additional uncertainty of 0.06% in the hadronic widths, corresponding to ± 1 MeV in Γ_Z. These predictions are to be compared with the experimental results, $\Gamma_Z = 2.4952 \pm 0.0023$ GeV [11] and $\Gamma_W = 2.085 \pm 0.042$ GeV [185] (see the Gauge & Higgs Boson Particle Listings for more details).

1.4. Precision flavor physics

In addition to cross-sections, asymmetries, parity violation, W and Z decays, there is a large number of experiments and observables testing the flavor structure of the SM. These are addressed elsewhere in this *Review*, and are generally not included in this Section. However, we identify three precision observables with sensitivity to similar types of new physics as the other processes discussed here. The branching fraction of the flavor changing transition $b \to s\gamma$ is of comparatively low precision, but since it is a loop-level process (in the SM) its sensitivity to new physics (and SM parameters, such as heavy quark masses) is enhanced. A discussion can be found in earlier editions of this *Review*. The τ-lepton lifetime and leptonic branching ratios are primarily sensitive to α_s and not affected significantly by many types of new physics. However, having an independent and reliable low energy measurement of α_s in a global analysis allows the comparison with the Z lineshape determination of α_s which shifts easily in the presence of new physics contributions. By far the most precise observable discussed here is the anomalous magnetic moment of the muon (the electron magnetic moment is measured to even greater precision and can be used to determine α, but its new physics sensitivity is suppressed by an additional factor of m_e^2/m_μ^2). Its combined experimental and theoretical uncertainty is comparable to typical new physics contributions.

The extraction of α_s from the τ lifetime [186] is standing out from other determinations because of a variety of independent reasons: (i) the τ-scale is low, so that upon extrapolation to the Z scale (where it can be compared to the theoretically clean Z lineshape determinations) the α_s error shrinks by about an order of magnitude; (ii) yet, this scale is high enough that perturbation theory and the operator product expansion (OPE) can be applied; (iii) these observables are fully inclusive and thus free of fragmentation and hadronization effects that would have to be modeled or measured; (iv) duality violation (DV) effects are most problematic near the branch cut but there they are suppressed by a double zero at $s = m_\tau^2$; (v) there are data [43] to constrain non-perturbative effects both within ($\delta_{D=6,8}$) and breaking (δ_{DV}) the OPE; (vi) a complete four-loop order QCD calculation is available [178]; (vii) large effects associated with the QCD β-function can be re-summed [187] in what has become known as contour improved perturbation theory (CIPT). However, while there is no doubt that CIPT shows faster convergence in the lower (calculable) orders, doubts have been cast on the method by the observation that at least in a specific model [188], which includes the exactly known coefficients and theoretical constraints on the large-order behavior, ordinary fixed order perturbation theory (FOPT) may nevertheless give a better approximation to the full result. We therefore use the expressions [54,177,178,189],

$$\tau_\tau = \hbar \frac{1 - \mathcal{B}_\tau^s}{\Gamma_\tau^e + \Gamma_\tau^\mu + \Gamma_\tau^{ud}} = 291.13 \pm 0.43 \text{ fs}, \tag{1.41}$$

$$\Gamma_\tau^{ud} = \frac{G_F^2 m_\tau^5 |V_{ud}|^2}{64\pi^3} S(m_\tau, M_Z) \left(1 + \frac{3}{5}\frac{m_\tau^2 - m_\mu^2}{M_W^2}\right) \times$$

$$\left[1 + \frac{\alpha_s\,(m_\tau)}{\pi} + 5.202\,\frac{\alpha_s^2}{\pi^2} + 26.37\,\frac{\alpha_s^3}{\pi^3} + 127.1\,\frac{\alpha_s^4}{\pi^4} + \frac{\widehat{\alpha}}{\pi}\left(\frac{85}{24} - \frac{\pi^2}{2}\right) + \delta_q\right], \qquad (1.42)$$

and Γ_τ^e and Γ_τ^μ can be taken from Eq. (1.6) with obvious replacements. The relative fraction of decays with $\Delta S = -1$, $\mathcal{B}_\tau^s = 0.0286 \pm 0.0007$, is based on experimental data since the value for the strange quark mass, $\widehat{m}_s(m_\tau)$, is not well known and the QCD expansion proportional to $\widehat{m}_s^2$ converges poorly and cannot be trusted. $S(m_\tau, M_Z) = 1.01907 \pm 0.0003$ is a logarithmically enhanced EW correction factor with higher orders re-summed [190]. δ_q contains the dimension six and eight terms in the OPE, as well as DV effects, $\delta_{D=6,8} + \delta_{DV} = -0.004 \pm 0.012$ [191]. Depending on how $\delta_{D=6}$, $\delta_{D=8}$, and δ_{DV} are extracted, there are strong correlations not only between them, but also with the gluon condensate ($D = 4$) and possibly $D > 8$ terms. These latter are suppressed in Eq. (1.42) by additional factors of α_s, but not so for more general weight functions. A simultaneous fit to all non-perturbative terms [191] (as is necessary if one wants to avoid *ad hoc* assumptions) indicates that the α_s errors may have been underestimated in the past. Higher statistics τ decay data [45] and spectral functions from e^+e^- annihilation (providing a larger fit window and thus more discriminatory power and smaller correlations) are likely to reduce the δ_q error in the future. Also included in δ_q are quark mass effects and the $D = 4$ condensate contributions. An uncertainty of similar size arises from the truncation of the FOPT series and is conservatively taken as the α_s^4 term (this is re-calculated in each call of the fits, leading to an α_s-dependent and thus asymmetric error) until a better understanding of the numerical differences between FOPT and CIPT has been gained. Our perturbative error covers almost the entire range from using CIPT to assuming that the nearly geometric series in Eq. (1.42) continues to higher orders. The experimental uncertainty in Eq. (1.41), is from the combination of the two leptonic branching ratios with the direct τ_τ. Included are also various smaller uncertainties (± 0.5 fs) from other sources which are dominated by the evolution from the Z scale. In total we obtain a $\sim 2\%$ determination of $\alpha_s(M_Z) = 0.1193^{+0.0022}_{-0.0020}$, which corresponds to $\alpha_s(m_\tau) = 0.327^{+0.019}_{-0.016}$, and updates the result of Refs. 54 and 192. For more details, see Refs. 191 and 193 where the τ spectral functions are used as additional input.

The world average of the muon anomalous magnetic moment‡,

$$a_\mu^{\rm exp} = \frac{g_\mu - 2}{2} = (1165920.80 \pm 0.63) \times 10^{-9}, \qquad (1.43)$$

‡ In what follows, we summarize the most important aspects of $g_\mu - 2$, and give some details about the evaluation in our fits. For more details see the dedicated contribution by A. Höcker and W. Marciano in this *Review*. There are some small numerical differences (at the level of 0.1 standard deviation), which are well understood and mostly arise because internal consistency of the fits requires the calculation of all observables from analytical expressions and common inputs and fit parameters, so that an independent evaluation is necessary for this Section. Note, that in the spirit of a global analysis based on all available information we have chosen here to average in the τ decay data, as well.

is dominated by the final result of the E821 collaboration at BNL [194]. The QED contribution has been calculated to four loops [195] (fully analytically to three loops [196,197]) , and the leading logarithms are included to five loops [198,199]. The estimated SM EW contribution [200–202], $a_\mu^{\rm EW} = (1.52 \pm 0.03) \times 10^{-9}$, which includes leading two-loop [201] and three-loop [202] corrections, is at the level of twice the current uncertainty.

The limiting factor in the interpretation of the result are the uncertainties from the two- and three-loop hadronic contribution. *E.g.,* Ref. 20 obtained the value $a_\mu^{\rm had} = (69.23 \pm 0.42) \times 10^{-9}$ which combines CMD-2 [47] and SND [48] $e^+e^- \to$ hadrons cross-section data with radiative return results from BaBar [50] and KLOE [51]. This value suggests a 3.6 σ discrepancy between Eq. (1.43) and the SM prediction. An alternative analysis [20] using τ decay data and isospin symmetry (CVC) yields $a_\mu^{\rm had} = (70.15 \pm 0.47) \times 10^{-9}$. This result implies a smaller conflict (2.4 σ) with Eq. (1.43). Thus, there is also a discrepancy between the spectral functions obtained from the two methods. For example, if one uses the e^+e^- data and CVC to predict the branching ratio for $\tau^- \to \nu_\tau \pi^- \pi^0$ decays [20] we obtain an average of $\mathcal{B}_{\rm CVC} = 24.93 \pm 0.13 \pm 0.22_{\rm CVC}$, while the average of the directly measured branching ratio yields 25.51 ± 0.09, which is 2.3 σ higher. It is important to understand the origin of this difference, but two observations point to the conclusion that at least some of it is experimental: (i) There is also a direct discrepancy of 1.9 σ between $\mathcal{B}_{\rm CVC}$ derived from BaBar (which is not inconsistent with τ decays) and KLOE. (ii) Isospin violating corrections have been studied in detail in Ref. 203 and found to be largely under control. The largest effect is due to higher-order EW corrections [204] but introduces a negligible uncertainty [190]. Nevertheless, $a_\mu^{\rm had}$ is often evaluated excluding the τ decay data arguing [205] that CVC breaking effects (*e.g.*, through a relatively large mass difference between the $\rho^\pm$ and ρ^0 vector mesons) may be larger than expected. (This may also be relevant [205] in the context of the NuTeV result discussed above.) Experimentally [45], this mass difference is indeed larger than expected, but then one would also expect a significant width difference which is contrary to observation [45]. Fortunately, due to the suppression at large s (from where the conflicts originate) these problems are less pronounced as far as $a_\mu^{\rm had}$ is concerned. In the following we view all differences in spectral functions as (systematic) fluctuations and average the results.

An additional uncertainty is induced by the hadronic three-loop light-by-light scattering contribution. Two recent and inherently different model calculations yield $a_\mu^{\rm LBLS} = (+1.36 \pm 0.25) \times 10^{-9}$ [206] and $a_\mu^{\rm LBLS} = +1.37^{+0.15}_{-0.27} \times 10^{-9}$ [207] which are higher than previous evaluations [208,209]. The sign of this effect is opposite [208] to the one quoted in the 2002 edition of this *Review*, and has subsequently been confirmed by two other groups [209]. There is also the upper bound $a_\mu^{\rm LBLS} < 1.59 \times 10^{-9}$ [207] but this requires an *ad hoc* assumption, too. The recent Ref. 210 quotes the value $a_\mu^{\rm LBLS} = (+1.05 \pm 0.26) \times 10^{-9}$, which we shift by 2×10^{-11} to account for the more accurate charm quark treatment of Ref. 207. We also increase the error to cover all evaluations, and we will use $a_\mu^{\rm LBLS} = (+1.07 \pm 0.32) \times 10^{-9}$ in the fits.

Other hadronic effects at three-loop order contribute [211] $a_\mu^{\rm had}(\alpha^3) = (-1.00 \pm 0.06) \times 10^{-9}$. Correlations with the two-loop hadronic contribution and with $\Delta\alpha(M_Z)$ (see Sec. 1.2) were considered in Ref. 197 which also contains analytic results for the perturbative QCD contribution.

Altogether, the SM prediction is

$$a_\mu^{\rm theory} = (1165918.41 \pm 0.48) \times 10^{-9} , \qquad (1.44)$$

where the error is from the hadronic uncertainties excluding parametric ones such as from α_s and the heavy quark masses. Using a correlation of about 84% from the data input to the vacuum polarization integrals [20], we estimate the correlation of the total (experimental plus theoretical) uncertainty in a_μ with $\Delta\alpha(M_Z)$ as 24%. The overall 3.0 σ discrepancy between the experimental and theoretical a_μ values could be due to fluctuations (the E821 result is statistics dominated) or underestimates of the theoretical uncertainties. On the other hand, $g_\mu - 2$ is also affected by many types of new physics, such as supersymmetric models with large $\tan\beta$ and moderately light superparticle masses [212]. Thus, the deviation could also arise from physics beyond the SM.

1.5. Global fit results

In this section we present the results of global fits to the experimental data discussed in Sec. 10.3–Sec. 1.4. For earlier analyses see Refs. 128 and 213.

Table 1.5: Principal SM fit result including mutual correlations (all masses in GeV). Note that $\widehat{m}_c(\widehat{m}_c)$ induces a significant uncertainty in the running of α beyond $\Delta\alpha^{(3)}_{\rm had}(1.8\ {\rm GeV})$ resulting in a relatively large correlation with M_H. Since this effect is proportional to the quark's electric charge squared it is much smaller for $\widehat{m}_b(\widehat{m}_b)$.

M_Z	91.1874 ± 0.0021	1.00	−0.01	0.00	0.00	−0.01	0.00	0.14
$\widehat{m}_t(\widehat{m}_t)$	163.71 ± 0.95	−0.01	1.00	0.01	−0.01	−0.15	0.00	0.31
$\widehat{m}_b(\widehat{m}_b)$	4.197 ± 0.025	0.00	0.01	1.00	0.24	−0.04	0.01	0.04
$\widehat{m}_c(\widehat{m}_c)$	$1.266^{+0.032}_{-0.040}$	0.00	−0.01	0.24	1.00	0.09	0.03	0.14
$\alpha_s(M_Z)$	0.1196 ± 0.0017	−0.01	−0.15	−0.04	0.09	1.00	−0.01	−0.05
$\Delta\alpha^{(3)}_{\rm had}(1.8\ {\rm GeV})$	0.00561 ± 0.00008	0.00	0.00	0.01	0.03	−0.01	1.00	−0.16
M_H	99^{+28}_{-23}	0.14	0.31	0.04	0.14	−0.05	−0.16	1.00

The values for m_t [56–58], M_W [172,214], neutrino scattering [96–104], the weak charges of the electron [131], cesium [138,139] and thallium [140], the muon anomalous magnetic moment [194], and the τ lifetime are listed in Table 1.3. Likewise, the principal Z pole observables can be found in Table 1.4 where the LEP 1 averages of the

Table 1.3: Principal non-Z pole observables, compared with the SM best fit predictions. The first M_W value is from the Tevatron [214] and the second one from LEP 2 [172]. e-DIS [129] and the ν-DIS constraints from CDHS [102], CHARM [103], and CCFR [104] are included, as well, but not shown in the Table. The world averages for $g_{V,A}^{\nu e}$ are dominated by the CHARM II [98] results, $g_V^{\nu e} = -0.035 \pm 0.017$ and $g_A^{\nu e} = -0.503 \pm 0.017$. The errors are the total (experimental plus theoretical) uncertainties. The τ_τ value is the τ lifetime world average computed by combining the direct measurements with values derived from the leptonic branching ratios [54]; in this case, the theory uncertainty is included in the SM prediction. In all other SM predictions, the uncertainty is from M_Z, M_H, m_t, m_b, m_c, $\widehat{\alpha}(M_Z)$, and α_s, and their correlations have been accounted for. The column denoted Pull gives the standard deviations for the principal fit with M_H free, while the column denoted Dev. (Deviation) is for $M_H = 124.5$ GeV [215] fixed.

Quantity	Value	Standard Model	Pull	Dev.
m_t [GeV]	173.4 ± 1.0	173.5 ± 1.0	-0.1	-0.3
M_W [GeV]	80.420 ± 0.031	80.381 ± 0.014	1.2	1.6
	80.376 ± 0.033		-0.2	0.2
$g_V^{\nu e}$	-0.040 ± 0.015	-0.0398 ± 0.0003	0.0	0.0
$g_A^{\nu e}$	-0.507 ± 0.014	-0.5064 ± 0.0001	0.0	0.0
$Q_W(e)$	-0.0403 ± 0.0053	-0.0474 ± 0.0005	1.3	1.3
$Q_W(\mathrm{Cs})$	-73.20 ± 0.35	-73.23 ± 0.02	0.1	0.1
$Q_W(\mathrm{Tl})$	-116.4 ± 3.6	-116.88 ± 0.03	0.1	0.1
τ_τ [fs]	291.13 ± 0.43	290.75 ± 2.51	0.1	0.1
$\frac{1}{2}(g_\mu - 2 - \frac{\alpha}{\pi})$	$(4511.07 \pm 0.77) \times 10^{-9}$	$(4508.70 \pm 0.09) \times 10^{-9}$	3.0	3.0

ALEPH, DELPHI, L3, and OPAL results include common systematic errors and correlations [11]. The heavy flavor results of LEP 1 and SLD are based on common inputs and correlated, as well [11]. Note that the values of $\Gamma(\ell^+\ell^-)$, $\Gamma(\mathrm{had})$, and $\Gamma(\mathrm{inv})$ are not independent of Γ_Z, the R_ℓ, and σ_{had} and that the SM errors in those latter are largely dominated by the uncertainty in α_s. Also shown in both Tables are the SM predictions for the values of M_Z, M_H, $\alpha_s(M_Z)$, $\Delta\alpha_{\mathrm{had}}^{(3)}$ and the heavy quark masses shown in Table 1.5. The predictions result from a global least-square (χ^2) fit to all data using the minimization package MINUIT [216] and the EW library GAPP [21]. In most cases, we treat all input errors (the uncertainties of the values) as Gaussian. The reason is not that we assume that theoretical and systematic errors are intrinsically bell-shaped (which they are not) but because in most cases the input errors are combinations of many different (including statistical) error sources, which should yield approximately Gaussian *combined* errors by the large number theorem. Thus, if either the statistical components dominate or there are many components of similar size. An exception is the theory dominated error on the τ lifetime, which we recalculate in each χ^2-function call since it depends

Table 1.4: Principal Z pole observables and their SM predictions (*cf.* Table 1.3). The first $\bar{s}_\ell^2(A_{FB}^{(0,q)})$ is the effective angle extracted from the hadronic charge asymmetry, the second is the combined value from DØ [167] and CDF [168], and the third is from CMS [171]. The three values of A_e are (i) from A_{LR} for hadronic final states [162]; (ii) from A_{LR} for leptonic final states and from polarized Bhabba scattering [164]; and (iii) from the angular distribution of the τ polarization at LEP 1. The two A_τ values are from SLD and the total τ polarization, respectively.

Quantity	Value	Standard Model	Pull	Dev.
M_Z [GeV]	91.1876 ± 0.0021	91.1874 ± 0.0021	0.1	0.0
Γ_Z [GeV]	2.4952 ± 0.0023	2.4961 ± 0.0010	−0.4	−0.2
$\Gamma(\text{had})$ [GeV]	1.7444 ± 0.0020	1.7426 ± 0.0010	—	—
$\Gamma(\text{inv})$ [MeV]	499.0 ± 1.5	501.69 ± 0.06	—	—
$\Gamma(\ell^+\ell^-)$ [MeV]	83.984 ± 0.086	84.005 ± 0.015	—	—
σ_{had}[nb]	41.541 ± 0.037	41.477 ± 0.009	1.7	1.7
R_e	20.804 ± 0.050	20.744 ± 0.011	1.2	1.3
R_μ	20.785 ± 0.033	20.744 ± 0.011	1.2	1.3
R_τ	20.764 ± 0.045	20.789 ± 0.011	−0.6	−0.5
R_b	0.21629 ± 0.00066	0.21576 ± 0.00004	0.8	0.8
R_c	0.1721 ± 0.0030	0.17227 ± 0.00004	−0.1	−0.1
$A_{FB}^{(0,e)}$	0.0145 ± 0.0025	0.01633 ± 0.00021	−0.7	−0.7
$A_{FB}^{(0,\mu)}$	0.0169 ± 0.0013		0.4	0.6
$A_{FB}^{(0,\tau)}$	0.0188 ± 0.0017		1.5	1.6
$A_{FB}^{(0,b)}$	0.0992 ± 0.0016	0.1034 ± 0.0007	−2.6	−2.3
$A_{FB}^{(0,c)}$	0.0707 ± 0.0035	0.0739 ± 0.0005	−0.9	−0.8
$A_{FB}^{(0,s)}$	0.0976 ± 0.0114	0.1035 ± 0.0007	−0.5	−0.5
$\bar{s}_\ell^2(A_{FB}^{(0,q)})$	0.2324 ± 0.0012	0.23146 ± 0.00012	0.8	0.7
	0.23200 ± 0.00076		0.7	0.6
	0.2287 ± 0.0032		−0.9	−0.9
A_e	0.15138 ± 0.00216	0.1475 ± 0.0010	1.8	2.1
	0.1544 ± 0.0060		1.1	1.3
	0.1498 ± 0.0049		0.5	0.6
A_μ	0.142 ± 0.015		−0.4	−0.3
A_τ	0.136 ± 0.015		−0.8	−0.7
	0.1439 ± 0.0043		−0.8	−0.7
A_b	0.923 ± 0.020	0.9348 ± 0.0001	−0.6	−0.6
A_c	0.670 ± 0.027	0.6680 ± 0.0004	0.1	0.1
A_s	0.895 ± 0.091	0.9357 ± 0.0001	−0.4	− 0.4

itself on α_s. Sizes and shapes of the output errors (the uncertainties of the predictions and the SM fit parameters) are fully determined by the fit, and 1 σ errors are defined to correspond to $\Delta\chi^2 = \chi^2 - \chi^2_{\text{min}} = 1$, and do not necessarily correspond to the 68.3% probability range or the 39.3% probability contour (for 2 parameters).

The agreement is generally very good. Despite the few discrepancies discussed in the following, the fit describes well the data with a

χ^2/d.o.f. = 45.0/42. The probability of a larger χ^2 is 35%. Only the final result for $g_\mu - 2$ from BNL and $A_{FB}^{(0,b)}$ from LEP 1 are currently showing large (3.0 σ and 2.6 σ) deviations. In addition, A_{LR}^0 (SLD) from hadronic final states differs by 1.8 σ. g_L^2 from NuTeV is nominally in conflict with the SM, as well, but the precise status is under investigation (see Sec. 10.3).

A_b can be extracted from $A_{FB}^{(0,b)}$ when $A_e = 0.1501 \pm 0.0016$ is taken from a fit to leptonic asymmetries (using lepton universality). The result, $A_b = 0.881 \pm 0.017$, is 3.2 σ below the SM prediction[§] and also 1.6 σ below $A_b = 0.923 \pm 0.020$ obtained from $A_{LR}^{FB}(b)$ at SLD. Thus, it appears that at least some of the problem in $A_{FB}^{(0,b)}$ is experimental. Note, however, that the uncertainty in $A_{FB}^{(0,b)}$ is strongly statistics dominated. The combined value, $A_b = 0.899 \pm 0.013$ deviates by 2.8 σ. It would be difficult to account for this 4.0% deviation by new physics that enters only at the level of radiative corrections since about a 20% correction to $\widehat{\kappa}_b$ would be necessary to account for the central value of A_b [217]. If this deviation is due to new physics, it is most likely of tree-level type affecting preferentially the third generation. Examples include the decay of a scalar neutrino resonance [218], mixing of the b quark with heavy exotics [219], and a heavy Z' with family-nonuniversal couplings [220,221]. It is difficult, however, to simultaneously account for R_b, which has been measured on the Z peak and off-peak [222] at LEP 1. An average of R_b measurements at LEP 2 at energies between 133 and 207 GeV is 2.1 σ below the SM prediction, while $A_{FB}^{(b)}$ (LEP 2) is 1.6 σ low [172].

The left-right asymmetry, $A_{LR}^0 = 0.15138 \pm 0.00216$ [162], based on all hadronic data from 1992–1998 differs 1.8 σ from the SM expectation of 0.1475 ± 0.0010. The combined value of $A_\ell = 0.1513 \pm 0.0021$ from SLD (using lepton-family universality and including correlations) is also 1.8 σ above the SM prediction; but there is experimental agreement between this SLD value and the LEP 1 value, $A_\ell = 0.1481 \pm 0.0027$, obtained from a fit to $A_{FB}^{(0,\ell)}$, $A_e(\mathcal{P}_\tau)$, and $A_\tau(\mathcal{P}_\tau)$, again assuming universality.

The observables in Table 1.3 and Table 1.4, as well as some other less precise observables, are used in the global fits described below. In all fits, the errors include full statistical, systematic, and theoretical uncertainties. The correlations on the LEP 1 lineshape and τ polarization, the LEP/SLD heavy flavor observables, the SLD lepton asymmetries, and the deep inelastic and ν-e scattering observables, are included. The theoretical correlations between $\Delta\alpha_{\rm had}^{(5)}$ and $g_\mu - 2$, and between the charm and bottom quark masses, are also accounted for.

The data allow a simultaneous determination of M_Z, M_H, m_t, and the strong coupling $\alpha_s(M_Z)$. ($\widehat{m}_c$, $\widehat{m}_b$, and $\Delta\alpha_{\rm had}^{(3)}$ are also allowed to float in the fits, subject to the theoretical constraints [19,54] described

[§] Alternatively, one can use $A_\ell = 0.1481 \pm 0.0027$, which is from LEP 1 alone and in excellent agreement with the SM, and obtain $A_b = 0.893 \pm 0.022$ which is 1.9 σ low. This illustrates that some of the discrepancy is related to the one in A_{LR}.

in Sec. 1.2. These are correlated with α_s.) α_s is determined mainly from R_ℓ, Γ_Z, $\sigma_{\rm had}$, and τ_τ and is only weakly correlated with the other variables. The global fit to all data, including the hadron collider average $m_t = 173.4 \pm 1.0$ GeV, yields the result in Table 1.5 (the $\overline{\rm MS}$ top quark mass given there corresponds to $m_t = 173.5 \pm 1.0$ GeV). The weak mixing angle is determined to

$$\widehat{s}^{\,2}_Z = 0.23116 \pm 0.00012, \qquad\qquad s^2_W = 0.22296 \pm 0.00028,$$

while the corresponding effective angle is related by Eq. (1.37), *i.e.*, $\overline{s}^2_\ell = 0.23146 \pm 0.00012$.

As described in Sec. 1.2 and the paragraph following Eq. (1.43) in Sec. 1.4, there is considerable stress in the experimental e^+e^- spectral functions and also conflict when these are compared with τ decay spectral functions. These are below or above the 2σ level (depending on what is actually compared) but not larger than the deviations of some other quantities entering our analyses. The number and size or these deviations are not inconsistent with what one would expect to happen as a result of random fluctuations. It is nevertheless instructive to study the effect of doubling the uncertainty in $\Delta\alpha^{(3)}_{\rm had}(1.8\ {\rm GeV}) = (55.50 \pm 0.78) \times 10^{-4}$, (see Sec. 1.2) on the extracted Higgs mass. The result, $M_H = 95^{+28}_{-23}$ GeV, demonstrates that the uncertainty in $\Delta\alpha_{\rm had}$ is currently of only secondary importance. Note also, that a shift of ± 0.0001 in $\Delta\alpha^{(3)}_{\rm had}(1.8\ {\rm GeV})$ corresponds to a shift of ∓ 5 GeV in M_H or about one fifth of its total uncertainty. The hadronic contribution to $\alpha(M_Z)$ is correlated with $g_\mu - 2$ (see Sec. 1.4). The measurement of the latter is higher than the SM prediction, and its inclusion in the fit favors a larger $\alpha(M_Z)$ and a lower M_H (currently by about 4 GeV).

The weak mixing angle can be determined from Z pole observables, M_W, and from a variety of neutral-current processes spanning a very wide Q^2 range. The results (for the older low energy neutral-current data see Refs. 128 and 213) shown in Table 1.7 of the full *Review* are in reasonable agreement with each other, indicating the quantitative success of the SM. The largest discrepancy is the value $\widehat{s}^{\,2}_Z = 0.23193 \pm 0.00028$ from the forward-backward asymmetries into bottom and charm quarks, which is 2.8 σ above the value 0.23116 ± 0.00012 from the global fit to all data. Similarly, $\widehat{s}^{\,2}_Z = 0.23067 \pm 0.00029$ from the SLD asymmetries (in both cases when combined with M_Z) is 1.8 σ low. The SLD result has the additional difficulty (within the SM) of implying very low and excluded [173] Higgs masses. This is also true for $\widehat{s}^{\,2}_Z = 0.23098 \pm 0.00022$ from M_W and M_Z and — as a consequence — for the global fit. We have therefore included in Table 1.3 and Table 1.4 an additional column (denoted Deviation) indicating the deviations if $M_H = 124.5$ GeV [215] is fixed.

The extracted Z pole value of $\alpha_s(M_Z)$ is based on a formula with negligible theoretical uncertainty if one assumes the exact validity of the SM. One should keep in mind, however, that this value, $\alpha_s(M_Z) = 0.1197 \pm 0.0028$, is very sensitive to such types of new physics as non-universal vertex corrections. In contrast, the value derived from τ decays, $\alpha_s(M_Z) = 0.1193^{+0.0022}_{-0.0020}$, is theory dominated but less sensitive to new physics. The two values are in remarkable

Table 1.6: Values of $\widehat{s}^2_Z$, s^2_W, α_s, and M_H [in GeV] for various (combinations of) observables. Unless indicated otherwise, the top quark mass, $m_t = 173.4 \pm 1.0$ GeV, is used as an additional constraint in the fits. The (†) symbol indicates a fixed parameter.

Data	$\widehat{s}^2_Z$	s^2_W	$\alpha_s(M_Z)$	M_H
All data	0.23116(12)	0.22295(28)	0.1196(17)	99^{+28}_{-23}
All indirect (no m_t)	0.23118(14)	0.22285(35)	0.1197(17)	134^{+144}_{-65}
Z pole (no m_t)	0.23121(17)	0.22318(60)	0.1197(28)	102^{+133}_{-51}
LEP 1 (no m_t)	0.23152(20)	0.22383(67)	0.1213(30)	191^{+266}_{-105}
SLD + M_Z	0.23067(28)	0.22204(54)	0.1185 (†)	39^{+31}_{-19}
$A^{(b,c)}_{FB}$ + M_Z	0.23193(28)	0.22494(76)	0.1185 (†)	444^{+300}_{-178}
M_W + M_Z	0.23098(22)	0.22262(47)	0.1185 (†)	75^{+39}_{-30}
M_Z	0.23124(5)	0.22318(13)	0.1185 (†)	124.5 (†)
$Q_W(e)$	0.2332(15)	0.2252(15)	0.1185 (†)	124.5 (†)
Q_W (APV)	0.2311(16)	0.2230(17)	0.1185 (†)	124.5 (†)
ν_μ-N DIS (isoscalar)	0.2332(39)	0.2251(39)	0.1185 (†)	124.5 (†)
Elastic $\nu_\mu(\overline{\nu}_\mu)$-$e$	0.2311(77)	0.2230(77)	0.1185 (†)	124.5 (†)
e-D DIS (SLAC)	0.222(18)	0.214(18)	0.1185 (†)	124.5 (†)
Elastic $\nu_\mu(\overline{\nu}_\mu)$-$p$	0.211(33)	0.203(33)	0.1185 (†)	124.5 (†)

agreement with each other. They are also in perfect agreement with the averages from jet-event shapes in e^+e^- annihilation (0.1172 ± 0.0037) and lattice simulations (0.1185 ± 0.0007), whereas the DIS average (0.1150 ± 0.0021) is somewhat lower. For more details, other determinations, and references, see our Section 9 on "Quantum Chromodynamics" in this *Review*.

One can also determine the radiative correction parameters Δr: from the global fit one obtains $\Delta r = 0.0352 \pm 0.0009$ and $\Delta\widehat{r}_W = 0.06945 \pm 0.00019$. M_W measurements [172,214] (when combined with M_Z) are equivalent to measurements of $\Delta r = 0.0342 \pm 0.0015$, which is 0.9 σ below the result from all other data, $\Delta r = 0.0358 \pm 0.0011$. Fig. 10.6 shows the 1 σ contours in the M_W-m_t plane from the direct and indirect determinations, as well as the combined 90% CL region. The indirect determination uses M_Z from LEP 1 as input, which is defined assuming an s-dependent decay width. M_W then corresponds to the s-dependent width definition, as well, and can be directly compared with the results from the Tevatron and LEP 2 which have been obtained using the same definition. The difference to a constant width definition is formally only of $\mathcal{O}(\alpha^2)$, but is strongly enhanced since the decay channels add up coherently. It is about 34 MeV for M_Z and 27 MeV for M_W. The residual difference between working consistently with one or the other definition is about 3 MeV, *i.e.*, of typical size for non-enhanced $\mathcal{O}(\alpha^2)$ corrections [74–77].

Most of the parameters relevant to ν-hadron, ν-e, e-hadron, and $e^- e^\pm$ processes are determined uniquely and precisely from the data in "model-independent" fits (*i.e.*, fits which allow for an arbitrary EW gauge theory). The values for the parameters defined in Eqs. (1.15)–(10.22) are given in Table 10.7 along with the predictions of the SM. The agreement is very good. (The ν-hadron results including the original NuTeV data can be found in the 2006 edition of this *Review*, and fits with modified NuTeV constraints in the 2008 and 2010 editions.) The off Z pole e^+e^- results are difficult to present in a model-independent way because Z propagator effects are non-negligible at TRISTAN, PETRA, PEP, and LEP 2 energies. However, assuming e-μ-τ universality, the low energy lepton asymmetries imply [159] $4\,(g_A^e)^2 = 0.99 \pm 0.05$, in good agreement with the SM prediction $\simeq 1$.

1.6. Constraints on new physics

The Z pole, W mass, and low energy data can be used to search for and set limits on deviations from the SM. We will mainly discuss the effects of exotic particles (with heavy masses $M_{\rm new} \gg M_Z$ in an expansion in $M_Z/M_{\rm new}$) on the gauge boson self-energies. (Brief remarks are made on new physics which is not of this type.) Most of the effects on precision measurements can be described by three gauge self-energy parameters S, T, and U. We will define these, as well as related parameters, such as ρ_0, ϵ_i, and $\widehat{\epsilon}_i$, to arise from new physics only. *I.e.*, they are equal to zero ($\rho_0 = 1$) exactly in the SM, and do not include any (loop induced) contributions that depend on m_t or M_H, which are treated separately. Our treatment differs from most of the original papers.

Many extensions of the SM can be described by the ρ_0 parameter,

$$\rho_0 \equiv \frac{M_W^2}{M_Z^2\,\widehat{c}_Z^{\,2}\,\widehat{\rho}}\,, \tag{1.45}$$

which describes new sources of SU(2) breaking that cannot be accounted for by the SM Higgs doublet or m_t effects. $\widehat{\rho}$ is calculated as in Eq. (1.12) assuming the validity of the SM. In the presence of $\rho_0 \neq 1$, Eq. (1.45) generalizes the second Eq. (1.12) while the first remains unchanged. Provided that the new physics which yields $\rho_0 \neq 1$ is a small perturbation which does not significantly affect the radiative corrections, ρ_0 can be regarded as a phenomenological parameter which multiplies G_F in Eqs. (1.15)–(10.22), (1.31), and Γ_Z in Eq. (1.38c). There are enough data to determine ρ_0, M_H, m_t, and α_s, simultaneously. From the global fit,

$$\rho_0 = 1.0004^{+0.0003}_{-0.0004}\,, \tag{1.46}$$

$$115.5 \text{ GeV} \le M_H \le 127 \text{ GeV}, \tag{1.47}$$

$$m_t = 173.4 \pm 1.0 \text{ GeV}, \tag{1.48}$$

$$\alpha_s\,(M_Z) = 0.1195 \pm 0.0017, \tag{1.49}$$

where the limits on M_H are nominal direct search bounds at the 95% CL [173,224,227]. In addition, the LHC is not yet sensitive to very large values of $M_H > 600$ GeV which are thus not ruled out either. In this very high mass scenario, we obtain,

$$\begin{aligned}
\rho_0 &= 1.0024^{+0.0010}_{-0.0003}\,, && (1.50)\\
0.6\text{ TeV} \le M_H &\le 1.2\text{ TeV}, && (1.51)\\
\alpha_s(M_Z) &= 0.1191 \pm 0.0016, && (1.52)
\end{aligned}$$

with the same m_t. Finally, if the direct search results are ignored entirely one finds $M_H = 189^{+568}_{-114}$ GeV and $\rho_0 = 1.0008^{+0.0020}_{-0.0011}$. The result in Eq. (1.46) is slightly above but consistent with the SM expectation, $\rho_0 = 1$. It can be used to constrain higher-dimensional Higgs representations to have vacuum expectation values of less than a few percent of those of the doublets. Indeed, the relation between M_W and M_Z is modified if there are Higgs multiplets with weak isospin $> 1/2$ with significant vacuum expectation values. For a general (charge-conserving) Higgs structure,

$$\rho_0 = \frac{\sum_i \left[t(i)\,(t(i)+1) - t_3(i)^2\right] |v_i|^2}{2\sum_i t_3(i)^2\,|v_i|^2}, \tag{1.53}$$

where v_i is the expectation value of the neutral component of a Higgs multiplet with weak isospin $t(i)$ and third component $t_3(i)$. In order to calculate to higher orders in such theories one must define a set of four fundamental renormalized parameters which one may conveniently choose to be α, G_F, M_Z, and M_W, since M_W and M_Z are directly measurable. Then $\hat{s}^2_Z$ and ρ_0 can be considered dependent parameters.

Eq. (1.46) can also be used to constrain other types of new physics. For example, non-degenerate multiplets of heavy fermions or scalars break the vector part of weak SU(2) and lead to a decrease in the value of M_Z/M_W. A non-degenerate SU(2) doublet $\binom{f_1}{f_2}$ yields a positive contribution to ρ_0 [228] of

$$\frac{C\,G_F}{8\sqrt{2}\pi^2}\Delta m^2, \tag{1.54}$$

where

$$\Delta m^2 \equiv m_1^2 + m_2^2 - \frac{4m_1^2 m_2^2}{m_1^2 - m_2^2}\ln\frac{m_1}{m_2} \ge (m_1 - m_2)^2\,, \tag{1.55}$$

and $C = 1$ (3) for color singlets (triplets). Thus, in the presence of such multiplets,

$$\rho_0 = 1 + \frac{3\,G_F}{8\sqrt{2}\pi^2}\sum_i \frac{C_i}{3}\,\Delta m_i^2\,, \tag{1.56}$$

where the sum includes fourth-family quark or lepton doublets, $\binom{t'}{b'}$ or $\binom{E^0}{E^-}$, right-handed (mirror) doublets, non-degenerate vector-like fermion doublets (with an extra factor of 2), and scalar doublets such as $\binom{\tilde{t}}{\tilde{b}}$ in Supersymmetry (in the absence of L–R mixing).

Eq. (1.46) taken together with Eq. (1.56) implies at the 95% CL,

$$\sum_i \frac{C_i}{3}\,\Delta m_i^2 \leq (52\ \mathrm{GeV})^2\,. \tag{1.57}$$

Non-degenerate multiplets usually imply $\rho_0 > 1$. Similarly, heavy Z' bosons decrease the prediction for M_Z due to mixing and generally lead to $\rho_0 > 1$ [229]. On the other hand, additional Higgs doublets which participate in spontaneous symmetry breaking [230] or heavy lepton doublets involving Majorana neutrinos [231], both of which have more complicated expressions, as well as the vacuum expectation values of Higgs triplets or higher-dimensional representations can contribute to ρ_0 with either sign. Allowing for the presence of heavy degenerate chiral multiplets (the S parameter, to be discussed below) affects the determination of ρ_0 from the data, at present leading to a larger value (for fixed M_H).

A number of authors [232–237] have considered the general effects on neutral-current and Z and W boson observables of various types of heavy (*i.e.*, $M_{\mathrm{new}} \gg M_Z$) physics which contribute to the W and Z self-energies but which do not have any direct coupling to the ordinary fermions. In addition to non-degenerate multiplets, which break the vector part of weak SU(2), these include heavy degenerate multiplets of chiral fermions which break the axial generators. The effects of one degenerate chiral doublet are small, but in Technicolor theories there may be many chiral doublets and therefore significant effects [232].

Such effects can be described by just three parameters, S, T, and U, at the (EW) one-loop level. (Three additional parameters are needed if the new physics scale is comparable to M_Z [238]. Further generalizations, including effects relevant to LEP 2, are described in Ref. 239.) T is proportional to the difference between the W and Z self-energies at $Q^2 = 0$ (*i.e.*, vector SU(2)-breaking), while S ($S+U$) is associated with the difference between the Z (W) self-energy at $Q^2 = M^2_{Z,W}$ and $Q^2 = 0$ (axial SU(2)-breaking). Denoting the contributions of new physics to the various self-energies by Π^{new}_{ij}, we have

$$\widehat{\alpha}(M_Z)\,T \equiv \frac{\Pi^{\mathrm{new}}_{WW}(0)}{M_W^2} - \frac{\Pi^{\mathrm{new}}_{ZZ}(0)}{M_Z^2}\,, \tag{1.58a}$$

$$\frac{\widehat{\alpha}(M_Z)}{4\,\widehat{s}^2_Z\widehat{c}^2_Z}S \equiv \frac{\Pi^{\mathrm{new}}_{ZZ}\left(M_Z^2\right) - \Pi^{\mathrm{new}}_{ZZ}(0)}{M_Z^2} - \frac{\widehat{c}^2_Z - \widehat{s}^2_Z}{\widehat{c}_Z\widehat{s}_Z}\,\frac{\Pi^{\mathrm{new}}_{Z\gamma}\left(M_Z^2\right)}{M_Z^2} - \frac{\Pi^{\mathrm{new}}_{\gamma\gamma}\left(M_Z^2\right)}{M_Z^2}\,, \tag{1.58b}$$

$$\frac{\widehat{\alpha}(M_Z)}{4\,\widehat{s}^2_Z}\,(S+U) \equiv \frac{\Pi^{\mathrm{new}}_{WW}\left(M_W^2\right) - \Pi^{\mathrm{new}}_{WW}(0)}{M_W^2} - \frac{\widehat{c}_Z}{\widehat{s}_Z}\,\frac{\Pi^{\mathrm{new}}_{Z\gamma}\left(M_Z^2\right)}{M_Z^2} - \frac{\Pi^{\mathrm{new}}_{\gamma\gamma}\left(M_Z^2\right)}{M_Z^2}\,. \tag{1.58c}$$

S, T, and U are defined with a factor proportional to $\widehat{\alpha}$ removed, so that they are expected to be of order unity in the presence of new physics. In the $\overline{\text{MS}}$ scheme as defined in Ref. 66, the last two terms in Eqs. (1.58*b*) and (1.58*c*) can be omitted (as was done in some earlier editions of this *Review*). These three parameters are related to other parameters (S_i, h_i, $\widehat{\epsilon}_i$) defined in Refs. [66,233,234] by

$$T = h_V = \widehat{\epsilon}_1/\widehat{\alpha}\,(M_Z)\,,$$
$$S = h_{AZ} = S_Z = 4\,\widehat{s}^2_Z\,\widehat{\epsilon}_3/\widehat{\alpha}\,(M_Z)\,,$$
$$U = h_{AW} - h_{AZ} = S_W - S_Z = -4\,\widehat{s}^2_Z\,\widehat{\epsilon}_2/\widehat{\alpha}\,(M_Z). \tag{1.59}$$

A heavy non-degenerate multiplet of fermions or scalars contributes positively to T as

$$\rho_0 - 1 = \frac{1}{1-\widehat{\alpha}\,(M_Z)\,T} - 1 \simeq \widehat{\alpha}\,(M_Z)\,T, \tag{1.60}$$

where ρ_0 is given in Eq. (1.56). The effects of non-standard Higgs representations cannot be separated from heavy non-degenerate multiplets unless the new physics has other consequences, such as vertex corrections. Most of the original papers defined T to include the effects of loops only. However, we will redefine T to include all new sources of SU(2) breaking, including non-standard Higgs, so that T and ρ_0 are equivalent by Eq. (1.60).

A multiplet of heavy degenerate chiral fermions yields

$$S = \frac{C}{3\pi}\sum_i \Big(t_{3L}\,(i) - t_{3R}\,(i)\Big)^2, \tag{1.61}$$

where $t_{3L,R}(i)$ is the third component of weak isospin of the left-(right-)handed component of fermion i and C is the number of colors. For example, a heavy degenerate ordinary or mirror family would contribute $2/3\pi$ to S. In Technicolor models with QCD-like dynamics, one expects [232] $S \sim 0.45$ for an iso-doublet of techni-fermions, assuming $N_{TC} = 4$ techni-colors, while $S \sim 1.62$ for a full techni-generation with $N_{TC} = 4$; T is harder to estimate because it is model-dependent. In these examples one has $S \geq 0$. However, the QCD-like models are excluded on other grounds (flavor changing neutral-currents, and too-light quarks and pseudo-Goldstone bosons [240]) . In particular, these estimates do not apply to models of walking Technicolor [240], for which S can be smaller or even negative [241]. Other situations in which $S < 0$, such as loops involving scalars or Majorana particles, are also possible [242]. The simplest origin of $S < 0$ would probably be an additional heavy Z' boson [229], which could mimic $S < 0$. Supersymmetric extensions of the SM generally give very small effects. See Refs. 243 and 244 and the note on “Supersymmetry” in the Searches Particle Listings for a complete set of references.

Most simple types of new physics yield $U = 0$, although there are counter-examples, such as the effects of anomalous triple gauge vertices [234].

The SM expressions for observables are replaced by

$$M_Z^2 = M_{Z0}^2\,\frac{1-\widehat{\alpha}\,(M_Z)\,T}{1-G_F M_{Z0}^2 S/2\sqrt{2}\pi}\ ,$$
$$M_W^2 = M_{W0}^2\,\frac{1}{1-G_F M_{W0}^2\,(S+U)\,/2\sqrt{2}\pi}\ , \tag{1.62}$$

where M_{Z0} and M_{W0} are the SM expressions (as functions of m_t and M_H) in the $\overline{\text{MS}}$ scheme. Furthermore,

$$\Gamma_Z = \frac{M_Z^3 \beta_Z}{1 - \widehat{\alpha}(M_Z)\, T}, \qquad \Gamma_W = M_W^3 \beta_W, \qquad A_i = \frac{A_{i0}}{1 - \widehat{\alpha}(M_Z)\, T}, \tag{1.63}$$

where β_Z and β_W are the SM expressions for the reduced widths Γ_{Z0}/M_{Z0}^3 and Γ_{W0}/M_{W0}^3, M_Z and M_W are the physical masses, and A_i (A_{i0}) is a neutral-current amplitude (in the SM).

The data allow a simultaneous determination of $\widehat{s}_Z^2$ (from the Z pole asymmetries), S (from M_Z), U (from M_W), T (mainly from Γ_Z), α_s (from R_ℓ, σ_{had}, and τ_τ), and m_t (from the hadron colliders), with little correlation among the SM parameters:

$$\begin{aligned} S &= 0.00^{+0.11}_{-0.10}, \\ T &= 0.02^{+0.11}_{-0.12}, \\ U &= 0.08 \pm 0.11, \end{aligned} \tag{1.64}$$

and $\widehat{s}_Z^2 = 0.23125 \pm 0.00016$, $\alpha_s(M_Z) = 0.1197 \pm 0.0018$, $m_t = 173.4 \pm 1.0$ GeV, where the uncertainties are from the inputs. We have used 115.5 GeV $< M_H <$ 127 GeV which is the allowed low mass window from LEP and the LHC. The SM parameters (U) can be determined with no (little) M_H dependence. On the other hand, S, T, and M_H cannot be obtained simultaneously from the precision data alone, because the Higgs boson loops themselves are resembled approximately by oblique effects. Negative (positive) contributions to the S (T) parameter can weaken or entirely remove the strong constraints on M_H from the SM fits. Specific models in which a large M_H is compensated by new physics are reviewed in Ref. 245. The parameters in Eqs. (1.64), which by definition are due to new physics only, are in reasonable agreement with the SM values of zero. Fixing $U = 0$ (as is also done in Fig. 10.7) moves S and T slightly upwards,

$$\begin{aligned} S &= 0.04 \pm 0.09, \\ T &= 0.07 \pm 0.08. \end{aligned} \tag{1.65}$$

The correlation between S and T in this fit amounts to 88%.

Using Eq. (1.60), the value of ρ_0 corresponding to T in Eq. (1.64) is 1.0001 ± 0.0009, while the one corresponding to Eq. (1.65) is $1.0005^{+0.0007}_{-0.0006}$. The values of the $\widehat{\epsilon}$ parameters defined in Eq. (1.59) are

$$\begin{aligned} \widehat{\epsilon}_3 &= 0.0000 \pm 0.0008, \\ \widehat{\epsilon}_1 &= 0.0001 \pm 0.0009 \\ \widehat{\epsilon}_2 &= -0.0006 \pm 0.0009. \end{aligned} \tag{1.66}$$

Unlike the original definition, we defined the quantities in Eqs. (1.66) to vanish identically in the absence of new physics and to correspond directly to the parameters S, T, and U in Eqs. (1.64). There is a strong correlation (89%) between the S and T parameters. The U parameter is -49% (-70%) anti-correlated with S (T). The allowed regions in S–T are shown in Fig. 10.7. From Eqs. (1.64) one obtains $S \leq 0.17$ and $T \leq 0.20$ at 95% CL.

A Section on constraints on new physics appears in the full *Review*.

Further discussion and all references may be found in the full *Review of Particle Physics;* the equation and reference numbering corresponds to that version.